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Formal theory of social creativity

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Abstract

This deliverable contains the beginnings of a formal theory of social creativity in which we give a social dimension to Goguen's Unified Concept Theory, a cognitive-discursive aspect to Barwise and Seligman's Information Flow and extend Lakatos's theory of concept change using concept blending. This paves the way for computational examples of social creativity from both mathematics and music.

Keyword list: **social creativity**, **collaboration**, **Unified Concept Theory**, **logic of distributed systems**, **Lakatos**

Changes

Version	Date	Author	Changes
0.1	23.09.15	Alison Pease	Send working draft to Joe
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Executive Summary

The aim of this work is create a formal model that is both descriptive of real world social creativity, and can be implemented computationally. The report centres on a prototype system that meets these criteria, but it also deals with treatments, that are on the one hand, more recognisably descriptive of real-world social creativity, but less formal; and, on the other, more computationally formal, but less directly recognisable. These efforts all have limitations, which we aim to address through integration.

Our main contribution to the goals of the deliverable is an implementation of Lakatos's dialectical patterns describing social creativity in mathematics. This research process begins with Lakatos's real-world examples which we have adapted into an abstract model of argumentation, and an accompanying computational implementation. We were then able to show how further real-world examples of collaborative mathematical proof map into this framework.

This is complemented by two additional strands of work. Firstly, we have analysed common processes found in successful collaborations, using the language of design patterns. This work is particularly useful for thinking about informal discourse that serves as "social glue." The degree to which social glue and out-of-domain discourse is required varies according to the scale and scope of a project. Secondly, we present a simple computational model of social creativity using cellular automata that have been modified to use Goguen's Unified Concept Theory to carry out evolutionary steps that evolve local behaviours over successive generations.

Work is underway to integrate these strands using a process calculus. In more complex domains, more different kinds of moves are needed. The structure of arguments is recursive, or more complex, even for relatively simple applications in the application domains of the Coinvent project. An argumentation-based approach may be useful in many cases but more complicated scenarios need more elaborate coordination methods.

Contents

1 Introduction

The aim of this work is create a formal model that is both descriptive of real world social creativity, and can be implemented computationally. The report centres on a prototype system that meets these criteria, but it also deals with treatments, that are on the one hand, more recognisably descriptive of real-world social creativity (but less formal), and, on the other, more computationally formal (but less directly recognisable). These efforts all have limitations, which we aim to address through integration. Figure 1 depicts the situation in graphical form:

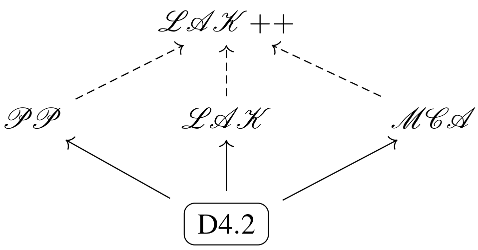

Figure 1: $\mathscr{LAK}$: Lakatosian creativity (Sections 2 and 3); $\mathscr{PP}$: Patterns of Peeragogy (Section 4); $\mathscr{MCA}$: Meta-Cellular Automata (Section 5); $\mathscr{LAK}$++: Integration effort (Section 6)

Our discussion of $\mathscr{LAK}$ is summarised in Section 2. Our work on the "recognisably social domain" $\mathscr{PP}$ is summarised in Section 4. Our work on the more formal domain $\mathscr{MCA}$ is summarised in Section 5. Some words about integration and steps towards $\mathscr{LAK}$++ are presented in Section 6. Full details can be found in [1, 7, 12, 2, 11], which we have highlighted in bold in the bibliography.

2 A formal representation of Lakatosian creativity

Lakatos describes a notion of social creativity in mathematics which is inherently based on blending two conflicting theories and synthesising a third theory from the inconsistencies [13]. Social aspects arise in that the conflicting theories come from different sources.

In [1], we explore the connections along the pipeline running from philosophical theory, to formal expression of dialogue games, extending earlier work in [2]. This allows us to express linguistic structures of reasoning in formal structured argumentation, abstract argumentation and argumentation semantics, and finally, coming full circle, we show that such implementations can provide value back to the linguistic communities from which the philosophical theory was derived. The goal is to show, for the first time, how all of these pieces of the puzzle can be slotted together to lay a foundation for formally sound and linguistically coherent collaborative intelligence and social creativity.

The interpretation of Lakatos here is through the lens of dialogue game theory [10] and in particular as a dialogue game ranging over structures of argumentation. The practice of interaction between mathematicians is mediated by language, so the choice of argumentation theory is, in the first instance, governed by the need to handle challenges presented by linguistic expressions of reasoning. On the other hand, in order to manipulate and automatically reason over those structures, there is a need for formal and ontological clarity, so we adopt the Argument Interchange

Format (AIF) [6, 17] to handle the data. The understanding of Lakatos is refined and characterised as a set of update semantics on AIF structures. Previous work [5] has shown how AIF can be interpreted as a nonmonotonic system of structured argumentation, with mappings built from AIF to one system in particular, ASPIC+[1] [16]. From ASPIC+, Prakken has shown how abstract argumentation systems in the style of [8] can be induced, from where the argumentation semantics can be computed to provide labellings of the acceptability status of each argument. Our report [1] shows how the labelling derived from the abstract argumentation framework corresponds precisely to the theory that has been collaboratively created by the participants in a Lakatosian dialogue.

In addition, however, each of these formal steps is also available in implementation. The interpretation of Lakatos as a formal dialogue game can be captured as an implemented specification in the domain specific language, the Dialogue Game Description Language (DGDL) [25]. This specification can be executed by a platform, the Dialogue Game Execution Platform (DGEP) [4] which offers a series of web services to clients for executing a participant's legal moves. Part of the semantics of the dialogue game specification is to define updates on a shared information state [24], in which the language of knowledge representation is AIF, implemented as a series of web services provided by the AIF database infrastructure [14]. The AIF data created as a side effect of the operational semantics of the turns in the dialogue game is in turn interpreted by The Online Argument Structures Tool (TOAST) [21] as an ASPIC+ system and passed to DungOMatic [20] to calculate the grounded extension, which returns, at each step in the game, the current state of the co-created theory.

Finally, we show how the model can also be retrospectively applied to examples of extant mathematical discussion [23]. In so doing, it is not only possible to demonstrate the depth of Lakatos' original insight, but also to show that the formal characterisation here remains both honest to the original and of practical utility to mathematicians. This ties into real-world social creativity in that firstly Lakatos's patterns were based on real world (if unusual and highly creative) examples, and secondly we show that, in implementation, the theory reflects the practices of working mathematicians holding everyday creative discussions.

This technical report constitutes our main contribution to the goals of the deliverable in that it is formal, descriptive of real world social creativity, and has been implemented.

3 Using Argumentation to Evaluate Concept Blends

The notion of value is crucial to computational creativity. In [7] we explore an argumentation approach to understanding and evaluating the meaning, interest, and significance of concept blends. Specifically, we propose viewing evaluating blends as a process of argumentation, in which the specifics of a blend are pinpointed and opened up as issues of discussion. This is based on the intuition that in the context of new ideas, proposals, or artworks, people use critical discussion and argumentation to understand, absorb and evaluate. We exemplify our approach in the domain of computer icon design, where icons are understood as creative artefacts generated through concept blending. We present a semiotic system for representing icons, showing how they can be described in terms of interpretations and how they are related by sign patterns. The interpretation of a sign

[1] ASPIC+ is the follow-on work from the Argumentation Service Platform with Integrated Components (ASPIC) project: http://www.cossac.org/projects/aspic

pattern conveys an intended meaning for an icon. This intended meaning is subjective, and depends on the way concept blending for creating the icon is realised. We show how the intended meaning of icons can be discussed in an explicit and social argumentation process modelled as a dialogue game, and show examples of these following the style of [1]. In this way, we are able to evaluate concept blends through an open-ended and dynamic discussion in which concept blends can be improved and the reasons behind a specific evaluation are made explicit. In the closing section, we explore argumentation and the potential roles that can play at different stages of the concept blending process.

This work concerns the notion of value and in particular the emergence of shared values, and the social processes by which people (dis)agree, explain and justify their evaluations. It relates more to real-world social creativity than formal theory.

4 Patterns of Peeragogy

"Patterns of Peeragogy" [11] uses the descriptive language of design patterns to take stock of the common processes found in successful collaborations, a typical context for everyday social creativity. This paper focuses on projects in which the structure of the collaboration is not fixed in advance and must be created alongside the project's primary products. The central pattern describes the ways in which project participants collaborate to build a shared ROADMAP that gets them "from *here* to *there*," possibly integrating multiple different "*heres* and *theres*." The nine design patterns introduced in the paper are particularly useful for thinking about informal discourse moves that serve as "social glue." For example, another pattern aims to address the needs of NEWCOMERS in a long-running project.

The degree to which social glue and out-of-domain discourse is required varies according to a project's scale and scope. For example, in the Polymath project, discussants needed to decide a large-scale overall plan – including which problems to focus on. On the other hand, in MiniPolymath, a single problem was set in advance and participants only needed to develop a suitable approach to solve it. Future work would formalise this material further, although the design pattern methodology already provides a preliminary functional breakdown.

5 The Search for Computational Intelligence

"The Search for Computational Intelligence" [12] aims to present a minimal convincing computational model of social creativity that exhibits emergent results. Previous research has viewed cellular automata "as multi-agent systems based on locality with overlapping interaction structures" [9]. Our paper takes the novel approach of permitting local adjustments to the behaviour of cells in the automaton. These new systems are called *meta-cellular automata* or "MetaCAs", since most cellular automata run according to one global rule, whereas MetaCAs co-evolve the local rule together with observed behaviour. From a philosophical perspective, this work is aligned with 20th Century philosopher George Mead's conception of the social as emergent co-evolution, "an adjustment in the organism and a reconstitution of the environment" [15]. Our simulations use a modified version of Goguen's Unified Concept Theory to carry out evolutionary steps that evolve local behaviours via concept blending over successive generations.

Expanding on the semantically simple domain of cellular automata, future work could encode mathematical problems and solution strategies in a MetaCA or "cellular program" and involve a group of agents in finding solutions to these problems as a society. This would be informed by the empirical analysis of real problem-solving activities, like MiniPolymath, as described in other sections of this report.

6 Integration Work

We have developed a process calculus analysis which attempts to structure all of the utterances in a mathematical conversation as a series of communications, using relations developed through analysis of the text. This work continues in the tradition of the Lightweight Social Calculus [19]. In particular, we are synthesising a protocol which can both account for the behaviour observed, and also admit the possibility of computational support. The underlying relations were developed both synthetically (by analysing samples of mathematical discourse), and through literature review. We presented an extended abstract describing this work at the First European Conference on Argumentation. This abstract is included as Appendix A.

Making the outer predicate one of a small set of illocutionary forces lets us look at protocols as sequences of locutions, which opens the way forward for dialogue induction. Opening the 'black box' of content – by adding structured identifiers – allows us to understand the relationships between posts, and based on this, we can make a picture of the whole interaction. For example, performatives are attached to text fragments, a provenance history of the evolution of core concepts in the discussion becomes available, and some segments may tagged as explicitly Lakatosian. Our strategy makes use of:

I. A (relatively) minimal set of performatives for the argumentation structure in the dialogue, such as *suggesting*, *asserting*, *retracting*, and *challenging*.

II. A set of relations between the mathematical structures under discussion, such as: *has_property*, *sub_conjecture*, *equivalent*, *stronger*, and *weaker*. This allows a model of *information flow* between structures, in the sense of [3].[2]

III. A set of meta objects representing the conjectures to prove, and strategies to use to prove them, such as *goal*, *strategy* and *difficult*.

Thus, for instance, we label the following utterance: "*The following reformulation of the problem may be useful: Show that for any permutation s in* S_n*, the sum* $a_s(1)+a_s(2)+a_s(j)$ *is not in M for any* $j =< n$*.*" [22] as *assert*(*equivalent*(main_problem,any_permutation)).

This outline reflects the view that much of an argument's structure is carried by relations between the mathematical objects under discussion. This strategy is useful, since we do not want to have to represent the whole of mathematics in order to reason about individual proofs: this gives us a level in between, and allows us to represent most of the important structure of specific arguments. Labelling individual snippets of conversation in this way enables us to construct an

[2]These relationships go beyond those explicitly dealt with in $\mathscr{LAK}$, and show recursive (and other) relationships between theories. Cf. the comparison with intuitionistic logic in [18].

overall picture of the interactions. We demonstrate this approach for the first few interactions in [22], in Figure 2, and the overall argument for further posts, in Figure 3.

Reflecting back over the work described in previous sections, the ways in which the method outlined in I–III could be expanded should be clear:

I′. As remarked in Section 4, in more complex domains, more kinds of moves are needed: for instance, problem positing as well as problem solving. We intend to progressively expand the approach outlined above to deal with more actively social problem scenarios, like Polymath.

II′. The ability to reason computationally about mathematical and musical structures is a core part of our plan of work in Coinvent. Therefore, we plan to integrate the type of agent-based model described in Section 5 with the simple structure annotations mentioned in II and CASL representations of mathematical and musical objects to create a convincing simulation of social creativity in our target domains.

III′. In this work, we are not necessarily restricted to a Lakatosian or even an argumentation-based approach, although these continue to be relevant in many situations. For example, in the work described in Section 3, we explore an argumentation-based approach to understanding and evaluating the meaning, interest, and significance of concept blends, using a simplified version of the Lakatosian dialogue game from [1]. However, scenarios with "multiple right answers" [11] require more elaborate coordination methods.

7 Outlook

We have reviewed our work this year on a formal theory of social creativity, which has largely taken place within the contexts of argumentation, social simulation, and design. We have furthermore summarised our plans for integration. The goal remains the creation of a formal, descriptive, and computationally implemented system. In the move from the Lakatosian theory $\mathscr{LAK}$ to "$\mathscr{LAK}$++", we have the opportunity to be both more descriptive of everyday social reality, and to explore more of what is computable using the techniques that Coinvent has to offer.

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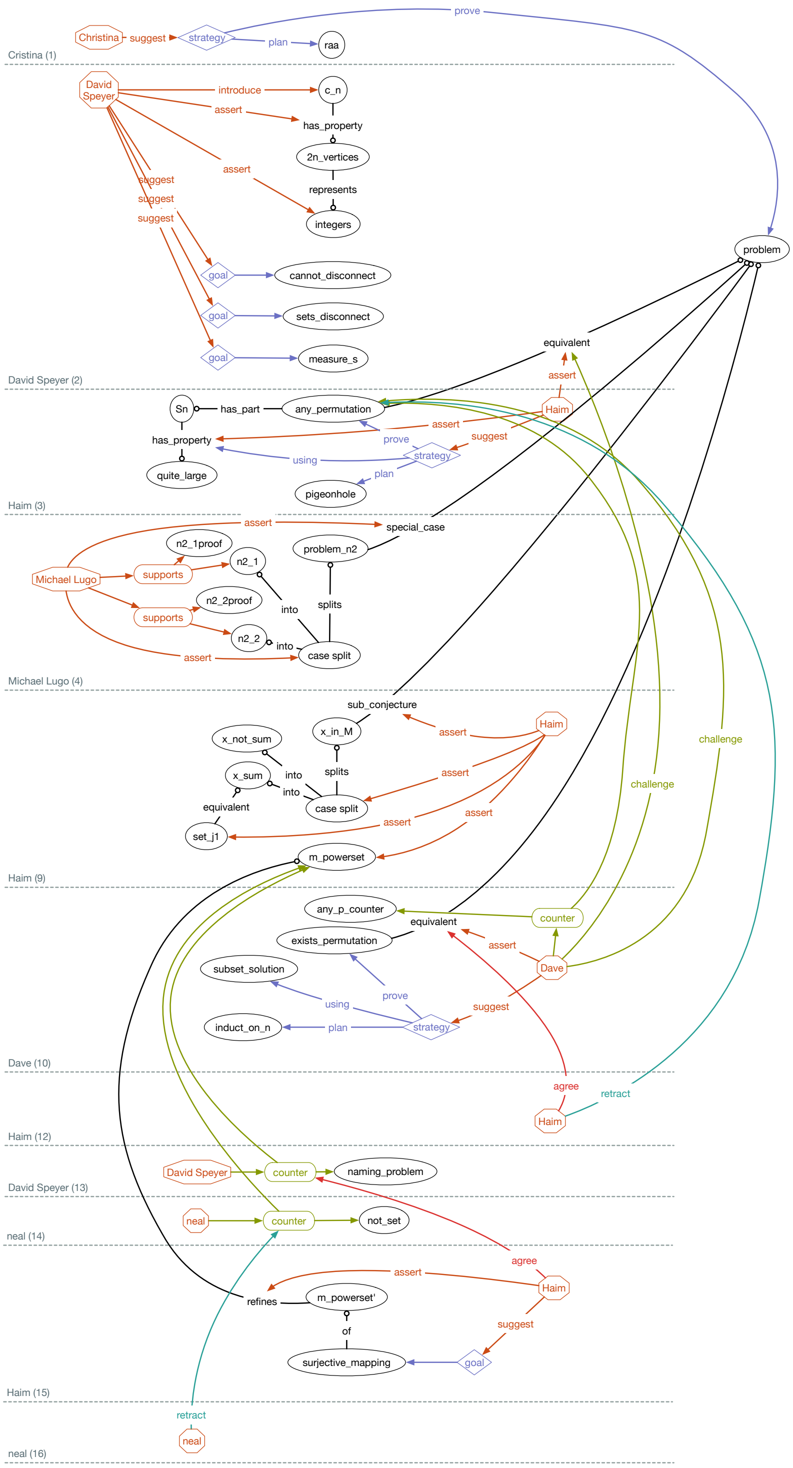

Figure 2: Interactions in MiniPolymath 1 [22], where the *person* making the utterance are in red; the *performatives* are in red (for supporting) and green (for challenging); the *mathematical objects* and relations between them are in black; and *meta-objects* are in purple. (image by David Murray-Rust, used with permission)

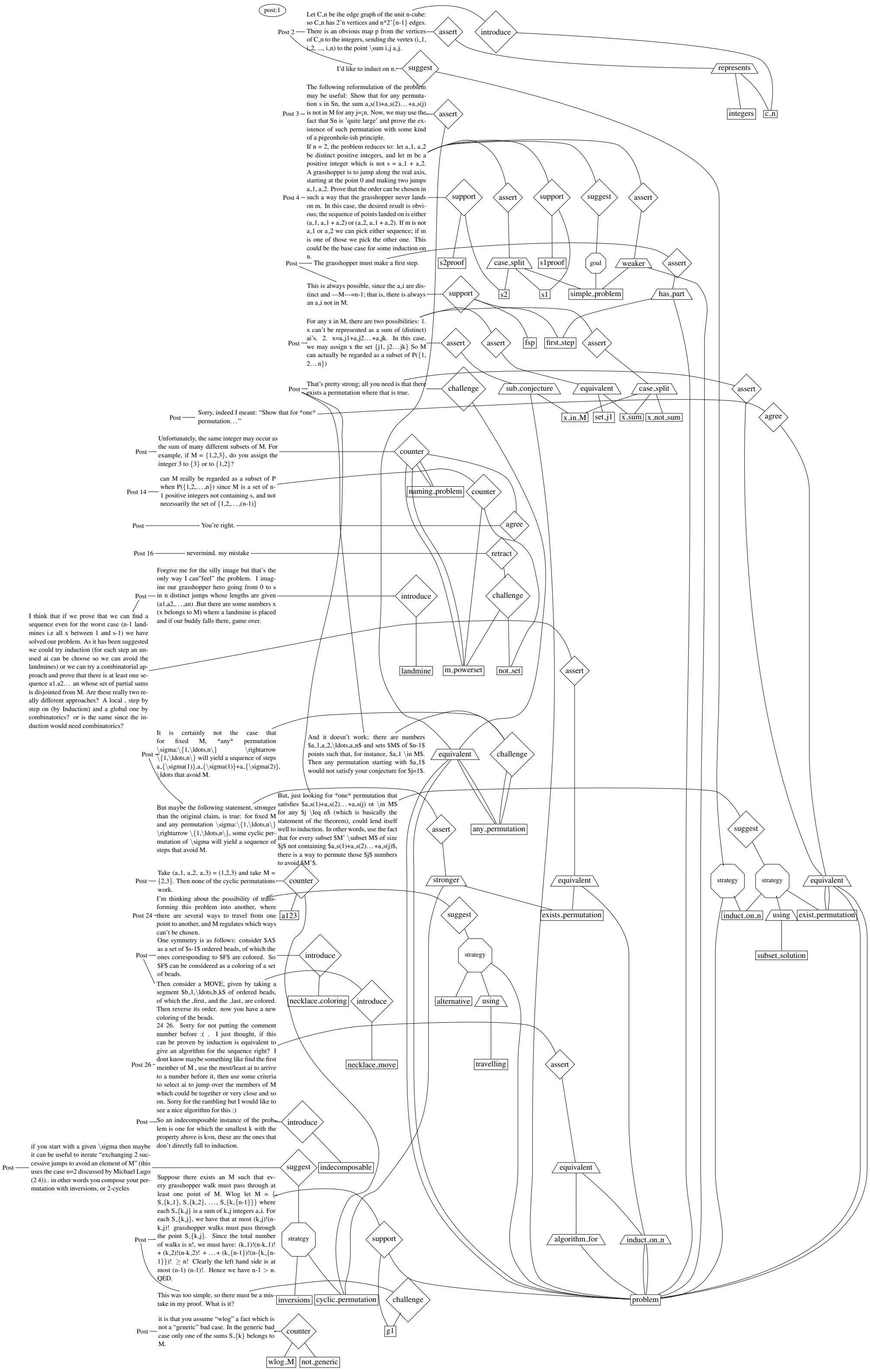

Figure 3: The argument in MP1 (image by David Murray-Rust, used and adapted with permission)

A Synchronised Multi-Perspective Analysis: ECA abstract

Synchronised multi-perspective analysis of online mathematical argument

1 A multi-perspective approach

Polymath [20] is an online experiment in collaborative mathematics. Participants work together to solve either open problems in mathematics, or questions from the International Mathematical Olympiad (IMO). The work is carried out online using a threaded discussion system for research and a wiki to represent the current state of knowledge.

This provides a compelling test-bed for developing theories of argumentation, as there is *(i)* a discourse where utterances are structured so that it is clear which post any comment relates to; *(ii)* a domain of discourse which can be formally represented; *(iii)* a synchronised representation of both how the dialogue unfolds over time and common knowledge which is a result of the dialogue; *(iv)* the potential for mechanical assistance to be brought in to the interaction if the activity can be sufficiently represented in computational terms.

From an argumentation perspective, there are several aspects of this interaction that can be analysed under different paradigms. In order to develop a rich understanding of how the process of doing mathematics collaborative is carried out, we analyse the same set of traces from multiple perspectives:

- A Lakatosian viewpoint [5], examining the utterances which fit into the schema of proofs and refutations, examples, counterexamples. Here we look only at those contributions which are congruent with Lakatos theories of how maths is done. This is an attempt to fit an existing structure to an example of observed argumentation.
- A process calculus analysis [8, 14] which attempts to structure all of the utterances as a series of communications, using relations developed through analysis of the text. Here, we try to synthesis a protocol which can both account for the behaviour observed, and through its formality admits the possibility of computational support.
- Inference Anchoring Theory (IAT) [2] which is developed through the interaction, which links into the utterances that provide components of the proof structure. This shows how the object under discussion relates to the text, highlighting the gaps in the structure due to shared context and implicitly understood reasoning.

Finally, we integrate these analyses around a temporal axis, and compare with the proof structures created to understand how the dynamic, sequential process of argumentation, of example and counterexample come together to form a coherent argument. This allows us to place the alternative perspectives alongside each other to demonstrate correspondences, areas of overlap and points of disagreement.

2 Related Work

Stahl has done extensive research on mathematics in a social context [17, 18, 19], developing research into "*group cognition*," drawing on online interactions between students. His notion of "*adjacency pairs*" is a broader category than what we call Lakatosian moves. Stahl's work focuses on computer supported collaborative learning, which is an inspiring domain for our work, although we also aim to add dimensions related to computer *simulated* collaborative learning. Nevertheless, our approach in the current phase of work, in which we will closely examine real-world dialogues, is similar to Stahl's. In our preliminary work, we have sought to establish a process coding suited to the social mathematics context. Earlier research focused on tagging texts created through speak-aloud protocals, for instance, Lucas et al. [7]. More recent work by Schoenfeld [15] situates mathematical problem solving in a social context, but the analysis of this material generally excludes social and cognitive details [15, p. 16]. Contemporary strategies from natural language processing, including the field of *argument mining* [11] which has gained recently traction within the broader field of *discourse mining* (e.g. [16, 21]) are also closely related, although here we only developing groundwork for future NLP-based efforts. In the current work we will not only look at argumentative structures, but also at pre-argumentative structures, that is, we consider the constructive features of *informal logic* (cf. [4]).

3 Illustrative Ideas

In Figures 1 and 2, we present short excerpts from MiniPolymath1 and MiniPolymath3, respectively, coded with tags drawn from the frameworks collected in Table 1. The tag sets `SA`, `AD`, `PD` are connected with primarily *dialogical* features; `MC`, `MO`, and `LD` with *logical* features; and `SS`, `PP`, `SF`, and `CE`, with *pragmatic* features. The first excerpt is adequately described as classic problem solving; it includes 6 "pragmatic" moves and 1 "dialogical" move. By contrast, the second is much more discursive, and includes 2 "pragmatic" moves, 3 "logical" moves, and 1 "dialogical" move. These distinctions are somewhat ad hoc; what is important to note is that earlier research has focused primarily on ("pragmatic") problem solving. When compared with the 42 primary tags used in [7], the tags that we used are much less focused on problem solving *per se*, although there is a significant overlap in the terminology and sources used. Our tag set will need to expand further in order to deal adequately with issues related to problem identification, positing, and selection.

NATE: Well, my first thought is to see if the hypotheses seem reasonable.[ss3] The hypothesis that $s = a_1 + \ldots + a_n$ not lie in M is certainly necessary, as the last jump that the grasshopper takes will land on s.[pp2] The grasshopper's other steps will land on a partial sums $a_{\sigma(1)} + \ldots + a_{\sigma(k)}$ for some permutation σ, but we get to choose the permutation. Thus it seems plausible that we can avoid a given set of $n-1$ points.[ss2]

[ss3] explore
[pp2] conditions
[ss2] analyze

THOMAS: Quick observation.[sa2] The grasshopper must make a first step.[sf2] This is always possible, since the a_i are distinct and $|M| = n-1$; that is, there is always an a_i not in M.[pp2] However, let's say M matches all but one of the a_i. Then the first step is uniquely determined. Still, according to the claimed theorem, a second step must still be possible.[pp3]

[sa2] inform
[sf2] heuristic (simplify)
[pp2] conditions
[pp3] decomposition

Figure 1: Excerpt from MiniPolymath1

HAGGAI NUCHI: The first point and line P_0, l_0 cannot be chosen[ce7] so that P_0 is on the boundary of the convex hull of S and l_0 picks out an adjacent point on the convex hull.[mc2] Maybe the strategy should be to take out the convex hull of S from consideration; follow it up by induction on removing successive convex hulls.[ld7]

[ce7] assertion
[mc2] example (monster)
[ld7] lemma incorporation

HAGGAI NUCHI: More specifically, remove the subset of S which forms the convex hull to get S_1; remove the new convex hull to get S_2, and repeat until S_n is convex. Maybe a point of S_n is a good place to start.[ss5]

[ss5] implement

SRIVATSAN NARAYANAN: Can we just assume by induction that we have proved the result for all the "inner points" $S_2 \cup S_3 \cdot \cup S_n$.[ad5] The base case would be that $S = S_1$, i.e., it forms a convex polygon.[mc4]

[ad5] negotiation
[mc4] proof

Figure 2: Excerpt from MiniPolymath3

`SA`	ackn, inform	*Speech-Act Annotated Corpus Classified List of Speech Acts* [6]
`AD`	negotiation	*Aberdein's patterns of proof dialogue* [1]
`PD`	question	*Prakken's persuasion dialogues* [13]
`LD`	conjecture, lemma incorporation	*Lakatosian moves* [9]
`MC`	concept, example (arbitrary inst.), example (monster), proof, other (phatic)	*types of mathematical comments* [10]
`MO`	article, problem, ephemera	*types of mathematical objects* [3]
`SS`	read, analyze, explore, implement, verify, local assessments, transition	*Schoenfeld's interpretation of "How to Solve It"* [15]
`PP`	goal, conditions, decomposition	*Pólya's stages of planning* [12]
`SF`	resources, heuristic (compute!), heuristic (decompose), heuristic (formal gen.), heuristic (simplify), heuristic (symmetry), heuristic (total stuckness), control, belief systems	*Schoenfeld's factors in mathematical thinking* [15]
`CE`	property, assertion	*components of explanation* [in preparation]

Table 1: Relevant tag sets with example tags

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